

UNIT - 2

SHEAR FORCE AND BENDING MOMENT

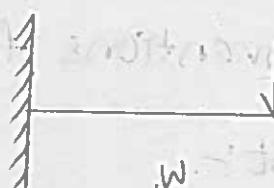
[Refr Strength of material - P.D.T. R.K. Bansal]

Definition of beam:- A structural member which is acted upon by a system of external load at right angles to its axis is known as beam. we see that whenever a horizontal beam is loaded with vertical loads, sometimes, it bends (i.e deflects). due to the action of the loads. the amount with which a beam bends depends upon the amount and type of the loads.

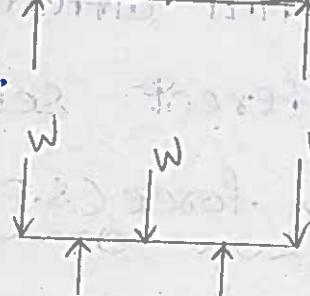
Types of beams:-

The following are the important types of beams.

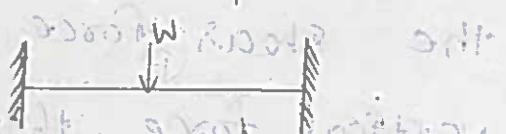
1. Cantilever beam



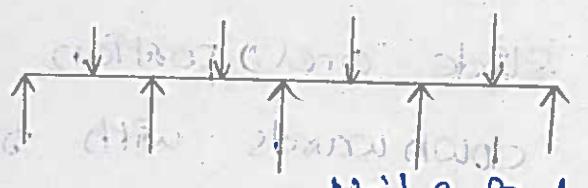
2. Simply supported beam.



3. Over hanging beam



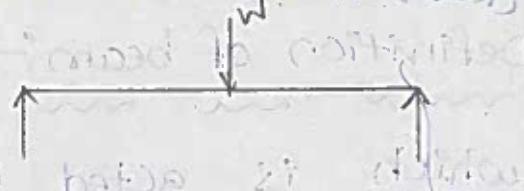
4. Rigidly fixed beam



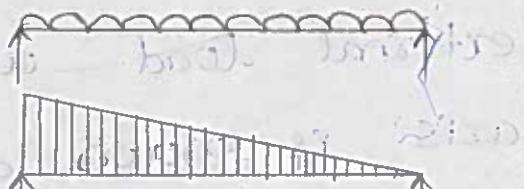
5. Continuous beam

Types of loadings:- A beam may be subjected to either or in combination of the following types of loads.

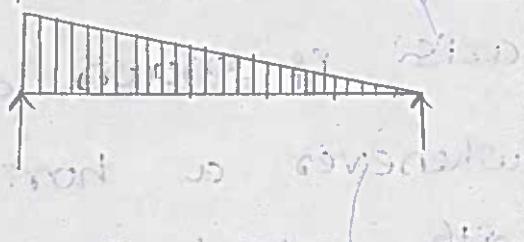
1. concentrated load



2. uniformly distributed load



3. uniformly varying load.



Concept of shear force & bending moment:

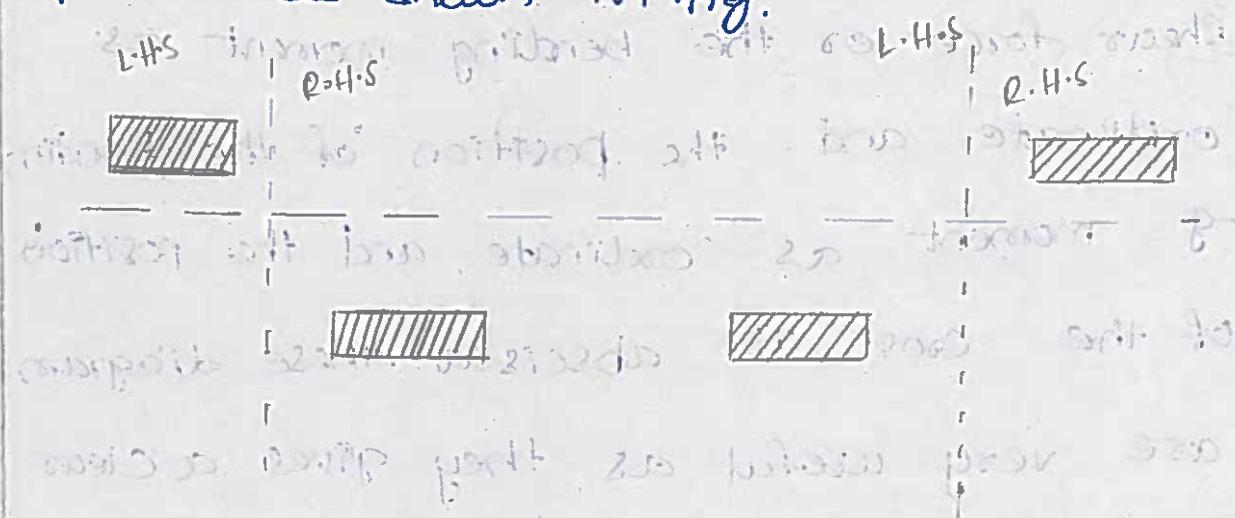
Shear force:- The shear force at the cross-section of a beam is defined as the algebraic sum of the forces to the right or left of the section (is denoted left B.M.)

Sign conventions for shear force & Bending moment:-

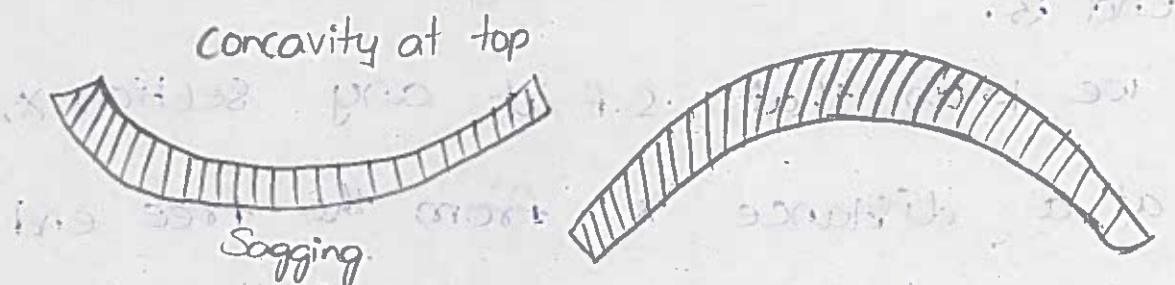
We find different sign conventions in different sections.

1. Shear force (S.F):- We know that as the shear force is the unbalanced vertical force, therefore, it tends to slide one portion of beam, upwards or downwards with respect to the other.

② we take shear force at a section as positive when the left hand portion tends to slide upwards (or) the right hand portion tends to slide down words. similarly, we take S.F at a section as negative when the left hand portion tends to slide downward (or) the right hand portion tends to slide upwards as shown in fig.



2. Bending moment :- we take bending moment (BM) at a section as positive, if it tends to bend the beam at that point, to a curvature having concavity at the top as shown in fig.



@ positive B.M.

Shear force and bending moment diagrams:
the shear force and bending moment can be calculated numerically at any particular section. But sometimes we are interested to know the manner in which these values vary along the length of the beam. This can be done by plotting the shear force or the bending moment as ordinate and the position of the bending moment as coordinate and the position of the coors as abscissa. These diagrams are very useful as they give a clear picture of the distribution of shear force and bending moment all along the beam.

Cantilever with a point load at its free end:

Fig shows a cantilever AB of length 'l' is fixed at 'A' and free at 'B' and carrying a point load 'w' at the free end 'B'.

We know that S.F at any section x, at a distance 'x' from the free end is equal to the total unbalanced vertical force.

Cantilever with a point load at its free end:

Fig shows a cantilever AB of length l is fixed at 'A' and free at 'B' and carrying a point load 'W' at the free end B.

We know that S.F at any section x , at a distance x from the free end is equal to the total unbalanced vertical force.

$$\therefore f_x = +w \quad [+ve \text{ sign due to weight}]$$

and bending moment at this section

$$M_x = -wx^2 \quad [-ve \text{ sign due to hogging}]$$

from the bending moment equation, it is clear that the B.M at any section is proportional to the distance of the beam.

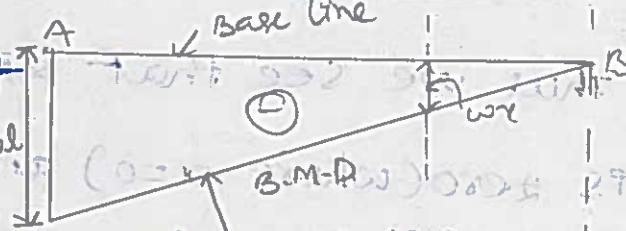
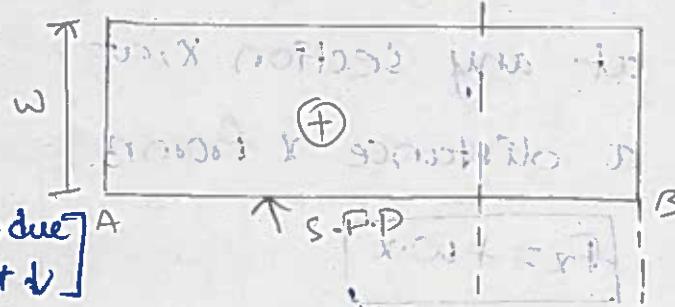
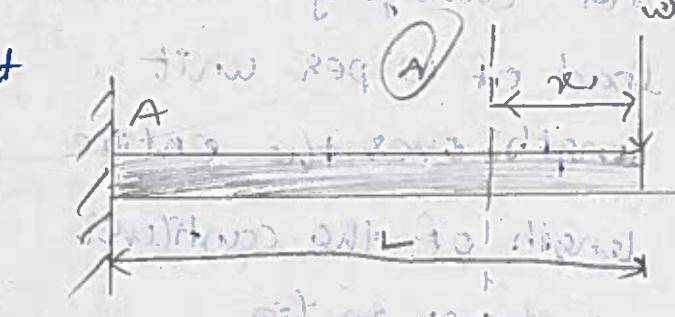
at $x=0$ 'c' at B

$$B.M_B = 0$$

at $x=L$ 'c' at A'

$$B.M_A = -WL$$

And from the B.M equation, we see that the bending moment is zero at B (where $x=0$) and increase by a straight line to $-WL$ at A (where $x=l$) as shown in fig.



Cantilever beam with a uniformly distributed load?

Consider a cantilever

beam AB of length l carrying a UDL

load of w per unit

length over the entire

length of the cantilever

as shown in fig.

We know that shear force

at any section x, at

a distance x from B.

$$F_x = +w \cdot x$$

thus we see that S.F

is zero (where $x=0$) and increases by a straight line law to $+wl$ at A as shown in fig.

We also know that the bending moment

at x.

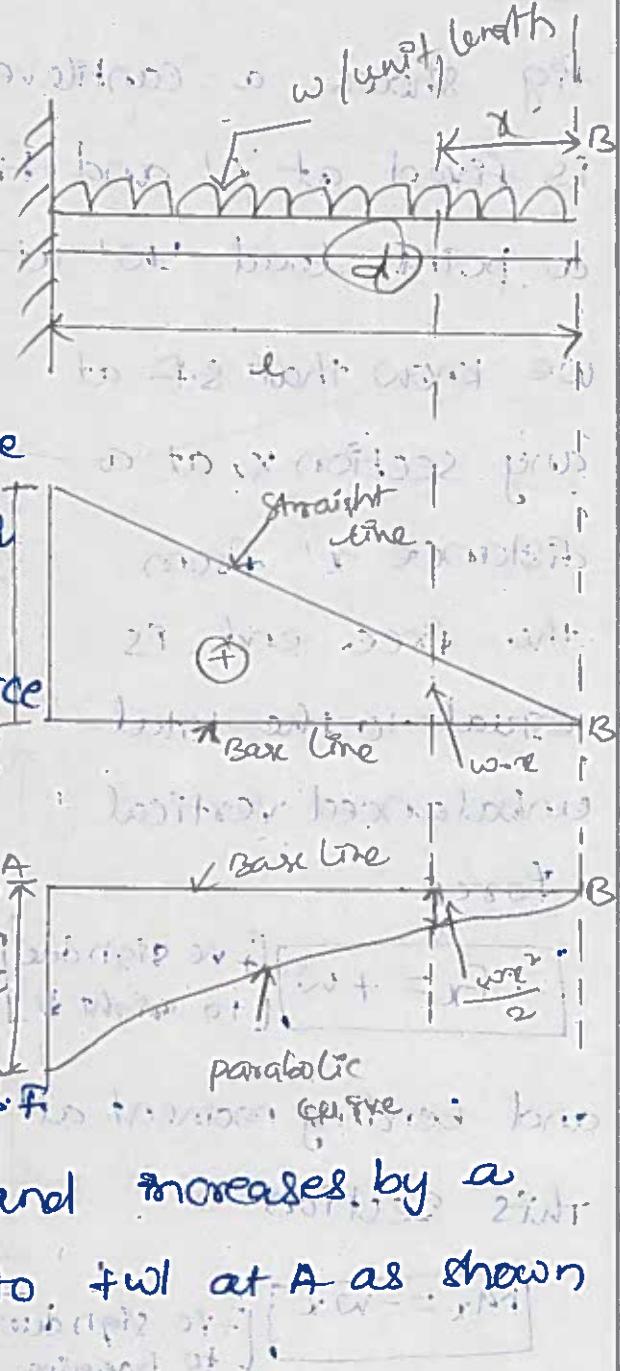
$$M_x = -wx \frac{x}{2} = -\frac{wx^2}{2}$$

thus we also see that B.M is zero at

B (where $x=0$) and increases in the

form of a parabolic curve to $\frac{-wl^2}{2}$

(where $x=l$) as shown in fig.



Cantilever with Gradually varying load :-

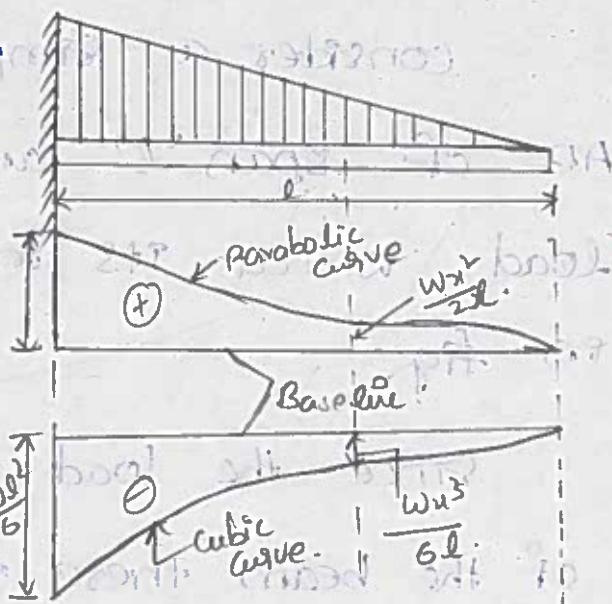
Consider a cantilever AB of length 'l'.

carrying a gradually

varying load from
zero at the free

end to w /unit length

at the fixed end, as $\frac{w}{6}x^2$
shown in fig.



We know that, the S.F
at any section x , at a
distance x from the
free end B.

$$\therefore F_x = + \frac{wx}{l} \cdot \frac{x}{2} = \frac{wx^2}{2l}$$

Thus, we see that the SF is zero at the free end (where $x=0$) and increases in the form of a parabolic curve $\frac{wx^2}{2l}$ as shown in fig.

We also know that the bending moment
at x

$$\therefore M_x = + \frac{wx^2}{2l} \times \frac{x}{3} = - \frac{wx^3}{6l}$$

thus we see that the bending moment
is zero at the free end (where $x=0$)
and increase in the form of a cubic curve
as shown in fig - $\frac{wx^3}{6l}$. Unit-II Pg No: 07/22

Simply Supported Beam with A point Load

At its Mid point:-

Consider a simply supported beam

AB of span 'l' and carrying a point load w at its mid-point 'c' as shown in fig.

Since the load is at the mid-point of the beam therefore from the reaction

at the support

$$f_A = f_B = \frac{w}{2}$$

Thus we see that the

at any section b/w

A and C (i.e up to

the point just

before the load w).

i.e constant and

is equal to the

unbalanced vertical

force + $\frac{w}{2}$. SF at

any section b/w C and B (i.e just

after the load w).

$$\therefore S.f_A = +\frac{w}{2}$$

$$S.f_B = -\frac{w}{2}$$

therefore bending moment at 'C'.

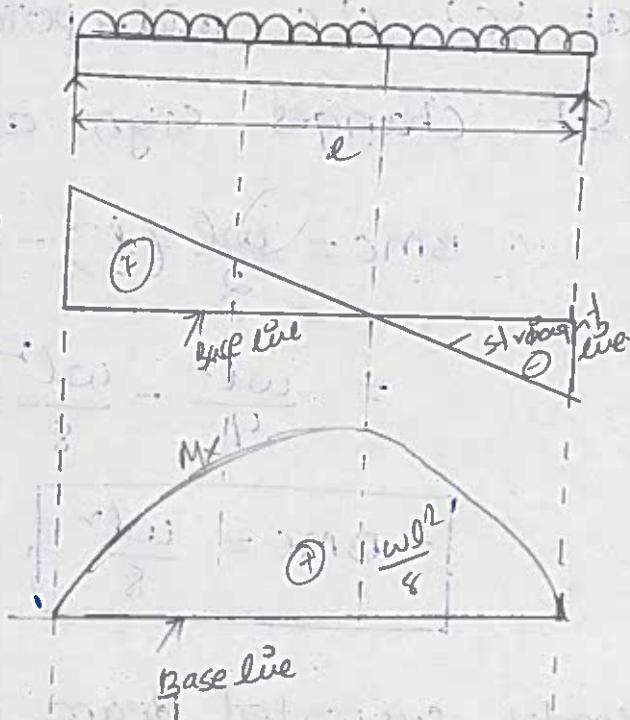
② $M_C = \frac{w}{2} \times \frac{l}{2} = \frac{wl}{4}$

$M_A = 0$

$M_B = 0$ due to reactions at A, B.

simply supported beam with uniformly distributed load:

Consider a simply supported beam AC of length l and carrying a UDL of w/unit as shown in fig.



∴ reactions at the supports

$$R_A = R_B = \frac{wl}{2}$$

we know that S.F at any section X at a distance 'x' from 'A'

$$F_x = R_A - w \cdot x = \frac{wl}{2} - wx.$$

we see that $S.F_A = \frac{wl}{2}$ where $x=0$.

$$S.F_B = + \frac{wl}{2} - wl = - \frac{wl}{2} \text{ where } x=l.$$

we also know that bending moment of any section at distance 'x' from A is

$$M_x = R_A x - \frac{wx^2}{2}$$

$$M_x = \frac{wlx}{2} - \frac{wx^2}{2}$$

We also see that bending moment is zero at A & B (where $x=0$ & $x=l$) and increases in the form of parabolic curve at 'C' C mid-point of the beam when S.F. changes sign as shown in fig.

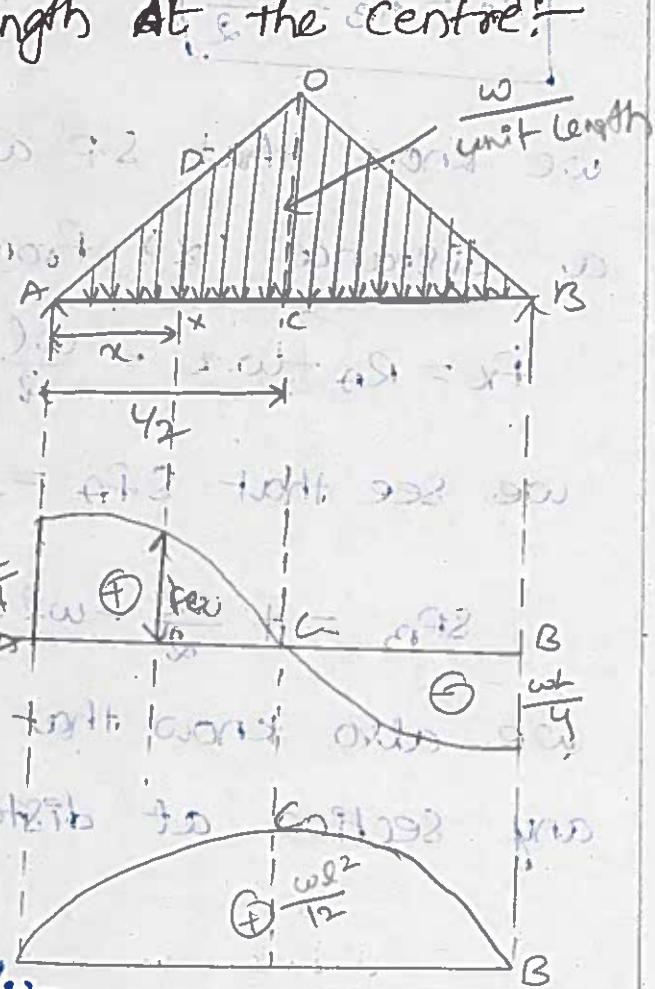
$$\therefore BMC = \frac{wl}{2} \cdot \left(\frac{l}{2}\right) - \frac{w}{2} \left(\frac{l}{2}\right)^2$$

$$= \frac{wl^2}{4} - \frac{wl^2}{8} = \frac{wl^2}{8}$$

$$\boxed{\therefore BMC = \frac{wl^2}{8}}$$

simply supported Beam with A Triangle Load, varying gradually from zero at Both Ends to w per unit length At the centre:-

fig shows a beam of length 'L' simply supported at the ends A & B and carrying a UDL from zero at each end to w /unit length at the centre.



at A, $x=0$ hence. $F_A = \frac{wL}{4} - 0 = \frac{wL}{4}$

at C; $x=\frac{L}{2}$ hence $F_C = \frac{wL}{4} - \frac{w}{L} \left[\frac{L}{2} \right]^2 = \frac{wL}{4} - \frac{wL}{4} = 0$

\therefore the shear force at B = $-f_B = f_B = -\frac{wL}{4}$

B.M diagram:

The bending moment is zero at A & B.

The B.M at x is given by

$$M_x = R_A x - \text{Load of length } Ax \left(\frac{x}{3} \right)$$

$$= \frac{wL}{4} x - \frac{w}{L} x^2 - \frac{x^3}{3}$$

$$= \frac{wLx}{4} - \frac{wx^3}{3L}$$

at A; $x=0$, $MA=0$

at C; $x=\frac{L}{2}$; $MC = \frac{wL}{4} \cdot \frac{L}{2} - \frac{w}{3L} \left(\frac{L}{2} \right)^3$

$$MC = \frac{wL^2}{8} - \frac{wL^2}{24} = \frac{wL^2}{12}$$

\therefore max B.M is at C = $MC = \frac{wL^2}{12}$

simply supported Beam carrying Uniformly varying Load from zero at one end to $w/\text{unit length}$ at the other end.

fig shows a beam AB of length 'L'

simply supported at the ends

\therefore Reactions at both ends

(3)

$$R_A = R_B = \frac{1}{2} \left(\frac{\omega L}{2} \right)$$

$$= \frac{\omega L}{4}$$

S.F at any section 'x' at a distance 'x' from B.

Consider only sections x b/w 'A' and 'c'.

\therefore = vertical distance XD in load diagram.

$$= \frac{x \cdot \omega}{4}$$

$$= \frac{2x \cdot \omega}{L}$$

Now load on the length Ax of the beam.

$$\Rightarrow \frac{x \cdot XD}{2}$$

$$= x \frac{2\omega}{L} x$$

$$= \frac{\omega}{L} x^2$$

This load is acting at a distance of $\frac{2}{3}$ from x.

Now S.F at x is given by

$$F_x = R_A - \text{load on the length Ax}$$

$$F_x = \frac{\omega L}{4} - \frac{\omega}{L} x^2$$

$$\therefore R_A = \frac{\omega L}{4}$$

⑥

'A' and 'B' and varying

load from zero at
end A to w per
unit length at B.

first calculate the

reactions R_A & R_B .

$$R_B \times L = \left(\frac{w \cdot L}{2}\right) \frac{2}{3} \frac{wL}{6}$$

$$R_B = \frac{w \cdot L}{3}$$

[total load $\left(\frac{wL}{2}\right)$ is
acting $\frac{2}{3}L$ from A]

and $R_A = \text{total load on beam} - \frac{wL}{3}$

$$R_A = \frac{w \cdot L}{2} - \frac{wL}{3} = \frac{wL}{6}$$

$$R_A = \frac{wL}{6}$$

and consider any section x at a distance
from end A.

$\& f_x = R_A - \text{load on length } Ax$

$$= \frac{wL}{6} + \frac{wx}{L} \cdot \frac{x}{2}$$

$$= \frac{wL}{6} + \frac{wx^2}{2L}$$

[\therefore load on $Ax = \frac{Ax^2}{2}$]

$$f_x = \frac{wL}{6} + \frac{wx^2}{2L}$$

$$= \frac{x \cdot w \cdot x}{2L}$$

from above equation

at A, $x=0$; $F_A = \frac{wL}{6}$, $\tau_0 = \frac{wL}{6}$

at B; $x=L$; $F_B = \frac{wL}{6} - \frac{wx^2}{2L} = \frac{wL}{6} - \frac{wL}{2} = -\frac{wL}{3}$

the S.F. is $\frac{wL}{6}$ at A. and it decreases to $-\frac{wL}{3}$ at B according to parabolic law.

Some where b/w A & B S.F. must be zero.

Now equating S.F. to zero in eqn (1) we get

$$0 = \frac{wL}{6} - \frac{wx^2}{2L}$$

$$\frac{wx^2}{2L} = \frac{wL}{6}$$

$$x^2 = \frac{wL}{G} \times \frac{2L}{w} = \frac{L^2}{3}$$

$$x = \frac{L}{\sqrt{3}} = 0.577L$$

B.M diagram

the B.M is zero at A and B.

The B.M of the section x at a distance x from the end A is given by

$$M_x = R_A x - \text{load on length } Ax \frac{x}{3}$$

$$M_x = \frac{wLx}{6} - \frac{wx^3}{6L}$$

-②

[\because load on Ax is acting at $\frac{1}{3}$ from x]

equation ② shows the B.M varies b/w A & B according to cubic law the point is at a distance $\frac{L}{\sqrt{3}}$ from A. Hence substituting

$$x = \frac{L}{\sqrt{3}}$$

$$\therefore M_{\min} = \frac{wL}{6} \cdot \frac{L}{\sqrt{3}} - \frac{w}{6L} \left(\frac{L}{\sqrt{3}}\right)^3$$

$$M_{\max} = \frac{wL^2}{6\sqrt{3}} - \frac{wL^2}{18\sqrt{3}} = \frac{wL^2}{9\sqrt{3}}$$

$$\therefore M_{\min} = \frac{wL^2}{9\sqrt{3}}$$

Shear Force And Moment Diagrams For Over-Hang Beam Length Equal Over Hinges And Carrying A UDL of $w/\text{unit area}$.

Shows a beam DABC of length $(l+2a)$ with supports at A & B

so that $AB=l$ and $DA=BC=a$. Let the beam carries a UDL of w per unit run over the entire length.

$$\therefore R_A = R_B = \frac{w(l+2a)}{2}$$

SF just to the right of B = $-wa$

SF just to the left of B

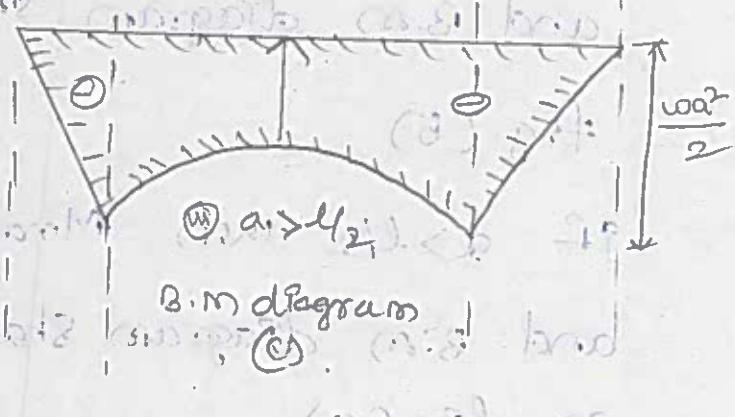
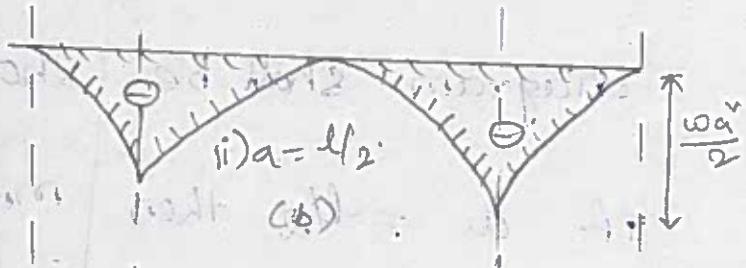
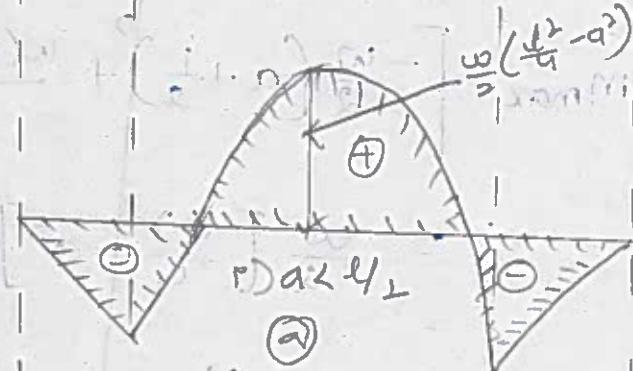
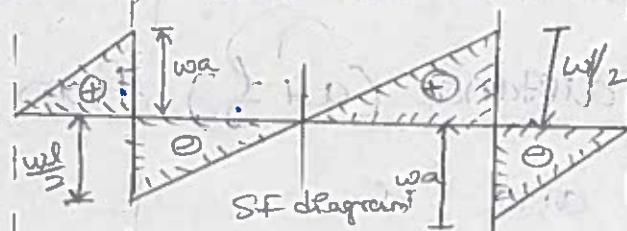
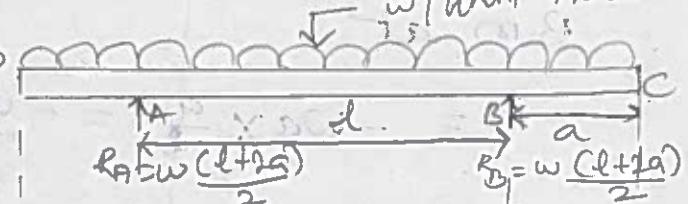
$$= -wa + \frac{w(l+2a)}{2}$$

$$= +\frac{wl}{2}$$

S.F. just to the right of A

$$= -w(l+a) + \frac{w(l+2a)}{2}$$

$$= -\frac{wl}{2}$$



SF just to the left of A

$$= -w(l+a) + 2 \times \frac{w(l+2a)}{2}$$
$$= +wa$$

the SF is $+wl/2$ at B and $-wl/2$ at A as such that it will be zero at the mid point between A and B.

B.M at

$$B = -wa \times \frac{a}{2} = -\frac{w a^2}{2}$$

B.M at the point of zero shear i.e. at a distance $(a + \frac{l}{2})$ from C. is the maximum and is

$$M_{max} = \left[-\frac{w}{2} \left(a + \frac{l}{2} \right)^2 + \frac{w(l+2a)}{2} \times \frac{l}{2} \right]$$
$$= \frac{w}{2} \left[\frac{l^2}{4} - a^2 \right]$$

$a < l/2$ then M_{max} will be +ve and B.M diagram shall be shown in fig(a)

If $a = l/2$ then M_{max} shall be zero and B.M diagram shall be shown in fig (b)

If $a > l/2$ then M_{max} shall be negative and B.M diagram shall be as shown in fig (c)

③ If B.M is zero at a distance "x" from either end then

$$= \frac{wx^2}{2} + \frac{wC(l+2a)}{2} (x-a) = 0$$

(0)

$$x = \frac{(l+2a) \pm \sqrt{l^2 - 4a^2}}{2}$$

point of contraflexure:- The bending moments of opposite nature always produce curvature of beams in opposite directions. In a beam if the bending moment changes sign at a point, the point itself having zero bending moment, the beam changes curvature at this point of zero bending moment and this point is called the point of contraflexure. So at a point of contraflexure the beam flexes in opposite direction. The point of contraflexure is called the point of inflection or a virtual hinge.

Loading and B.M Diagrams from S.O.F Diagram

If the S.I. diagram for a beam is given (if the B.M diagram is given).

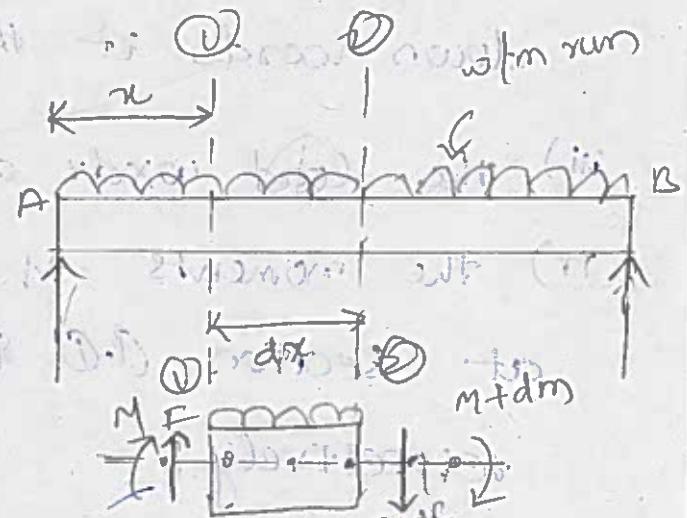
the loading diagrams for the beam can very easily be drawn, if the following point are remembered.

1. the S.F diagram will consist of rectangle, if the beam is loaded with point loads.
2. the S.F diagram will consist of inclined lines for the portion on which U.D.L is acting.
3. the S.F diagram will consist of parabolic lines for the portion over which triangular or trapezium load distribution is acting.
4. the S.R diagram will consist of 'cubics' for the portion over which parabolic load distribution is acting.
5. the B.M diagram will consist of inclined lines if the beam is loaded with free point loads.
6. the B.M diagram will consist of parabolic lines for portion over which U.D.L is acting.
7. the B.M diagram will consist of 'cubic' or third degree polynomials if the load distribution is a triangle.

- ⑨ 8. The B.M. diagrams will consist of fourth degree polynomial if the load distribution is parabolic.

Relation Between Load, Shear Force And Bending Moment

fig shows a beam carrying a uniformly distributed load w per unit length



Consider the equilibrium of the portion of the beam between sections 1-1 and 2-2. This portion is at a distance of x' from left support and is of length dx .

let F = shear force at the section 1-1

$F+df$ = shear force at the section 2-2

M = Bending moment at the section 1-1

$M+dm$ = Bending moment at the section 2-2

The forces and moment acting on the length ' dx ' of the beam are:

The force and moments on the beam

are

- i) the force F acting vertically up at the section (1) - (1)
- ii) the force $F + dF$ acting vertically down words at the section (2) - (2)
- iii) the load $wx dx$ acting down words.
- iv) the moments M and $(M + dm)$ acting at section (1) & section (2) - (2) respectively.

The portion of the beam of length dx is in equilibrium. Hence resolving the forces acting on this part vertically, we get:

$$F - w \cdot dx - (F + dF) = 0$$

$$-dF = w \cdot dx$$

$$\boxed{\frac{dF}{dx} = -w}$$

The above equation shows that

the rate of change of shear force is, acting equal to the rate of loading.

Taking the moments of the forces and couples about the section (2) - (1) we get.

$$M - w \cdot dx \frac{dx}{2} + F \cdot dx = M \cdot dm$$

$$-w \frac{dx^2}{2} + F \cdot dx = dm$$

(10) neglecting the higher powers of small quantity we get

$$F \cdot dx = dm$$

neglecting above the higher powers of small quantity we get

$$F \cdot dx = dm$$

$$F = \frac{dm}{dx} \text{ (or)} \frac{dm}{dx} = F$$

The above equation shows that the rate of change of bending moment is equal to the shear force at the section.

$$C_{\text{total}} = C_b \cdot A + \frac{C_b}{A} \cdot \Delta h \cdot A$$

$$C_{\text{total}} = C_b \cdot A + \frac{C_b}{A} \cdot \Delta h \cdot A$$

Now we can calculate the pressure

$$\rightarrow P = \rho g \cdot h_{\text{total}}$$

$$P = \rho g \cdot h_{\text{total}}$$

to calculate height we have to calculate

$$h_{\text{total}} = h_{\text{bottom}} + h_{\text{top}}$$

$$h_{\text{top}} = x \cdot h_{\text{bottom}}$$

$$\Rightarrow \frac{h_{\text{top}}}{h_{\text{bottom}}} = \frac{x \cdot h_{\text{bottom}}}{h_{\text{bottom}}} = x$$

Now we have exactly enough values to

calculate the pressure given to equation 15

and we will do this now with the help of